# Section 4: Systems of Equations

The following maps the videos in this section to the Texas Essential Knowledge and Skills for Mathematics TAC §111.42(c).

**4.01 Solving Systems of Equations: Substitution and Graphing**

- Precalculus (c)(1)(D)

**4.02 Solving Systems of Equations: Elimination**

- Precalculus (c)(1)(D)

**4.03 Solving Systems of Equations: Word Problems**

- Precalculus (c)(1)(A)
- Precalculus (c)(1)(B)
- Precalculus (c)(1)(C)

Note: Unless stated otherwise, any sample data is fictitious and used solely for the purpose of instruction.

# 4.01

# Solving Systems of Equations: Substitution and Graphing

***System of equations*** – A list of equations that are to be solved simultaneously

***Linear system*** – A system of equations consisting of linear equations

***Solution*** – An ordered pair that satisfies each equation in the system

***Solving the system of equations*** – Finding the set of all the solutions to the system

| Type of Linear System | Number of Solutions | Graphical Interpretation |
|---|---|---|
| Independent | One | Lines intersect at one point |
| Dependent | Infinite | Same line |
| Inconsistent | Zero | Parallel lines |

**Three Methods for Solving a System of Equations**

1. Substitution
2. Graphing
3. Elimination

**Substitution Method**

1. Solve one of the equations for one variable in terms of the other variable.
2. Substitute the expression found in Step 1 as the variable in the other equation to obtain an equation with a single variable.
3. Solve the equation created in Step 2.
4. Substitute this value obtained in Step 3 back into the equation found in Step 1.
5. Check that the solution satisfies each of the original equations by evaluating the solution in both equations.

Note: The substitution method outlined above works for nonlinear as well as linear equations.

1. Solve by substitution: $\begin{aligned} y &= 3x - 11 \\ y &= 5x + 7 \end{aligned}$

2. Solve the system $\begin{matrix} x^2 + y^2 = 25 \\ x + y = 1 \end{matrix}$ using the substitution method.

**Graphing Method**

***Point of intersection*** – A coordinate where two graphs intersect

The solution(s) to a system of equations can be found as the point(s) of intersection of two graphs.

3. Solve the system $\begin{matrix} y = \ln x \\ x + y = 1 \end{matrix}$ by graphing.

# 4.02

# Solving Systems of Equations: Elimination

**Elimination Method**

1. Manipulate the equations so that the coefficients of one variable differ only in sign.
2. Add the two equations to eliminate one variable.
3. Solve the equation created in Step 2.
4. Substitute the value obtained in Step 3 back into either of the original equations.
5. Check that the solution satisfies each of the original equations.

**What to look for when solving a linear system by elimination**

- An independent system will have a unique solution.
- A dependent system will reduce (when solving), leaving $0 = 0$.
- An inconsistent system will reduce (when solving) into an equality that is false, such as $0 = 5$.

1. Solve by elimination and determine the type of system: $\frac{1}{2}x + \frac{2}{3}y = 2$
$4x - 3y = -\frac{2}{3}$

2. Solve by elimination and determine the type of system: $\begin{aligned} 4x+10y&=11 \\ 6x+15y&=5 \end{aligned}$

3. Solve by elimination and determine the type of system: $\begin{aligned} 4x - 6y &= 8 \\ 6x - 9y &= 12 \end{aligned}$

# 4.03

## Solving Systems of Equations: Word Problems

1. Suppose you are tasked with designing a rectangular garden that has a perimeter of 400 feet. The dimensions must be set so that the length of the garden is 30 feet longer than the width. Set up a system of linear equations that describes the dimensions of the rectangular garden, and solve.

2. How much 20% alcohol solution needs to be added to a solution that is 50% alcohol to obtain 6 liters of 30% alcohol solution?

3. Suppose you invest $5,000 in stocks and bonds. Stocks return 8% in interest each year, while bonds return only 3%. If the interest after one year is $300, how much did you invest in stocks and bonds respectively?