Section 1: Introduction to Functions and Graphs

The following maps the videos in this section to the Texas Essential Knowledge and Skills for Mathematics TAC §111.42(c).

1.01 Lines
- Precalculus (1)(A)
- Precalculus (1)(B)
- Precalculus (1)(C)

1.02 Functions
- Precalculus (1)(A)
- Precalculus (1)(B)

1.03 Algebra of Functions
- Precalculus (2)(A)
- Precalculus (2)(B)
- Precalculus (2)(C)

1.04 Inverse Functions
- Precalculus (2)(E)

1.05 Graphs of Functions
- Precalculus (2)(D)
- Precalculus (2)(F)
- Precalculus (2)(G)
- Precalculus (2)(I)

Note: Unless stated otherwise, any sample data is fictitious and used solely for the purpose of instruction.

1.01

Lines

Properties of Lines

Slope –

Intercepts –

- To find the *x*-intercepts, set _______ and solve for _______.
- To find the *y*-intercepts, set _______ and solve for _______.

Parallel Lines – Have the same slope ($m_1 = m_2$)

Perpendicular Lines – Have the negative reciprocal ($m_1 = -\frac{1}{m_2}$)

Equations of Lines

- Slope-Intercept Form: $y = mx + b$
    - slope = m
    - *y*-intercept = b
- Point-Slope Form: $y - y_1 = m(x - x_1)$
    - slope = m
    - point = (x_1, y_1)
- General Form: $Ax + By = C$
    - slope = $-\frac{A}{B}$
- Vertical Line: $x = a$
    - slope: undefined
    - has an *x*-intercept
- Horizontal Line: $y = b$
    - slope = 0
    - has a *y*-intercept

1. Find the equation of a line that passes through the point $(4,1)$ and is perpendicular to the line that contains the points $(3,-7)$ and $(-1,9)$.

2. Find the equation of a line that is perpendicular to the line $x = 4$ and passes through the point $(3,11)$.

3. The freezing point of water is 0°C and 32°F, while the boiling point is 100°C and 212°F.

   i. Express the Fahrenheit temperature *F* in terms of the Celsius temperature *C*.

   ii. Determine the temperature in degrees Fahrenheit that corresponds to 42°C.

1.02
Functions

Function – a rule that assigns to every input value only one output value

- No two ordered pairs will contain the same first coordinate.
- Graphically, every function must pass the ______________________________.
- $f(x) = y$
  - This means that if $f(2) = 3$, the coordinate is ___________.
  - The coordinate $\left(x, f(x)\right)$ is equivalent to (x, y).
  - x is known as the independent variable, and ____________ is the set of all possible x-values.
  - y is known as the dependent variable, and ____________ is the set of all possible y-values.

1. Which of the following equations (relations) are functions?

i. $y = 3x + 4$ ii. $y = |x|$ iii. $|y| = x$

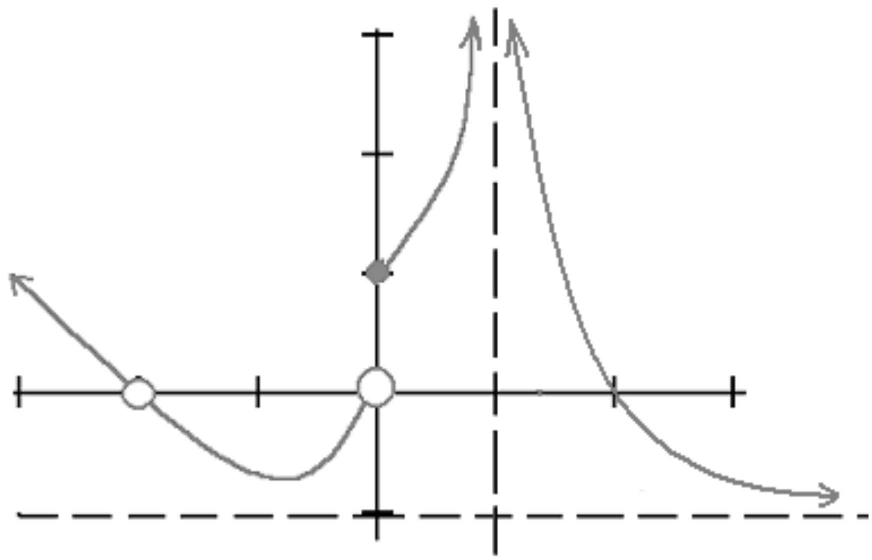
iv. $y = 3$ v. $x = 5$ vi. $y^2 = x + 3$

vii. $x = 3y + 1$ viii. $\{(2,3), (4,5), (6,6), (7,6)\}$ ix. $\{(1,1), (2,2), (2,1), (1,2)\}$

STUDY EDGE TIP

- In general, an equation is a function if the output variable has an odd power or root.
- An equation cannot be a function if the output variable has an even power or an absolute value.

2. Given the graph of $f(x)$ below, answer the following questions:



i. What is $f(0)$?

ii. For what value(s) of x does $f(x) = 0$?

iii. What are the domain and range of $f(x)$?

3. Given $f(x) = x^2 + 2x - 7$

i. Find $f(-2)$.

ii. Find the x-value where $f(x) = 1$.

iii. Evaluate $\frac{f(a+h) - f(a)}{h}$.

STUDY EDGE TIP

When asked to evaluate the expression $\frac{f(a+h) - f(a)}{h}$, the goal is to reduce the h in the denominator while simplifying. Common techniques include factoring, common denominators, and conjugates.

Domain – the input values of a given equation, also known as the set of all *x*-values

There are three main domain restrictions until you reach trigonometry.

Rule 1: A function cannot have a negative number under an even root. (You can have a zero.)

- Set what is inside the radical ≥ 0 and solve. The answer will be the domain.
- Find the domain of the following function: $f(x) = \sqrt{7-x}$

Rule 2: A function cannot have a zero in the denominator.

- Set the denominator = 0 and solve. Those are the numbers that are **<u>not</u>** in the domain.
- Find the domain of the following function: $f(x) = \dfrac{x+1}{(x+3)(x-1)}$

Rule 3: A function cannot have a negative or zero inside a log/ln.

- Set the inside or the argument > 0 and solve. The answer will be the domain.
- Find the domain of the following function: $f(x) = \ln\left(x^2 - 9\right)$

4. Find the domain of the following functions:

$f(x) = \frac{1}{\sqrt{3-x}}$ $\quad$ $f(x) = x^2 + 2x - 7$ $\quad$ $f(x) = \sqrt{1 - \frac{9}{x^2}}$

5. Suppose you are selling hotdogs at a hotdog stand in downtown Austin. You start selling hotdogs at $4 each and end up selling an average of 200 hotdogs per day. When you increase the price by $1, sales decrease by 20 hotdogs per day. Assume the relationship between hotdogs (*x*) and price (*p*) is linear.

   i. Determine two points in the form (x, p).

   ii. Calculate the slope of the linear function between hotdogs and price.

   iii. Find the equation of the line *p* as a function of *x*.

1.03
Algebra of Functions

Addition: $(f+g)(x)=f(x)+g(x)$

Subtraction: $(f-g)(x)=f(x)-g(x)$

Multiplication: $(fg)(x)=f(x)g(x)$

Division: $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0$

To find domain of the above functions, take into account the individual functions *and* the combined function.

Composition: $(f\circ g)(x)=f\big(g(x)\big)$

To find the domain of a composite function:

1) Determine the domain of the inner function.
2) Then, determine the domain of the composite function.
3) Lastly, combine steps 1 and 2 to find the overall domain of the composite function.

1. Let $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{x-4}$

   i. Evaluate the expression $(f+g)(9)$.

   ii. Evaluate the expression $(g\circ f)(9)$.

   iii. Find the function $(fg)(x)$ and its domain.

iv. Find the function $(f \circ g)(x)$ and its domain.

v. Find the function $(g \circ f)(x)$ and its domain.

2. Represent $y = (x - 2)^3 + 1$ as a composition of the two functions $f(x)$ and $g(x)$.

3. The cost, C, of buying a given product can be determined using the function $C(x) = 2x + 5$, where x is the amount of the product available. The availability of the product can be determined using $x(t) = 80 - 2t$ where $0 < t < 32$. Determine the formula for cost, based on the day of the month.

1.04
Inverse Functions

Properties of Inverse Functions

- The inverse of $f(x)$ is denoted $f^{-1}(x)$.
- For the inverse relation to be a function, it must pass the horizontal line test (and already pass the vertical line test because they are functions).
- Functions and their inverse relations are symmetric over the line $y = x$, meaning if $f(2) = 3$, then ____________.
- Functions can be one-to-one, meaning each x-value and y-value is used only once.
- If a function is one-to-one, then the inverse relation is also a function.
- To find the inverse function, switch x and y for the entire equation and then solve for y.
- If a function has an inverse, then $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.
- Sometimes if an inverse relation is not a function, then its domain can be restricted to make it a function.

1. Let $f(x) = (x-2)^3 + 1$

   i. Find $f^{-1}(x)$.

   ii. Show that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

2. Let $f(x) = \frac{2x-1}{x+5}$

   i. Find $f^{-1}(x)$.

   ii. Find $f^{-1}(3)$.

   iii. Find the domain and range of $f(x)$ and $f^{-1}(x)$.

STUDY EDGE TIP

- The domain of $f^{-1}(x)$ = the range of $f(x)$
- The range of $f^{-1}(x)$ = the domain of $f(x)$

3. Which of the functions below are one-to-one?

i. $f(x) = |x|$

ii. $f(x) = x^3$

iii. $f(x) = \frac{1}{x^2}$

iv. $f(x) = \frac{1}{x}$

v. $f(x) = \sqrt[3]{x}$

vi. $f(x) = \sqrt[4]{x}$

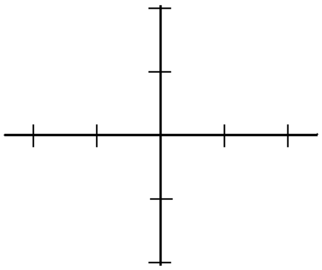
vii. $f(x) = x^2 - 4, x \geq 0$

STUDY EDGE TIP

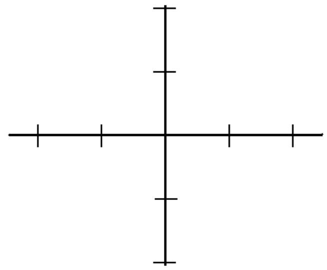
- If a function has an even power or absolute value for any variable, and it has no domain restriction, it will *not* be one-to-one.
- That does not mean that odd powers will necessarily be one-to-one. For example: $f(x) = x^3 - x$.

1.05
Graphs of Functions, Symmetry, Reflections, and Transformations

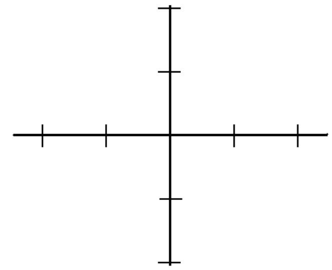
$f(x) = x^2$



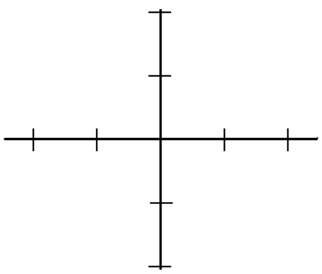
$f(x) = |x|$



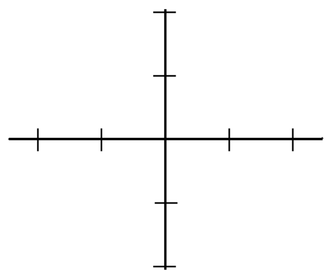
$f(x) = \sqrt{x}$



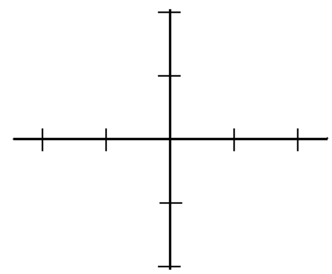
$f(x) = x^3$



$f(x) = \frac{1}{x}$



$f(x) = \sqrt[3]{x}$



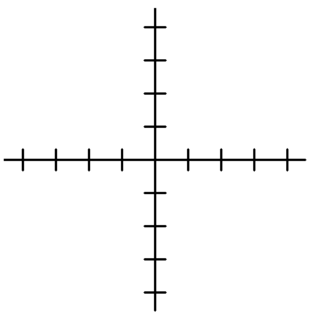
Given a function $f(x)$, and a constant $c > 0$:

$f(x) + c$	Shifts $f(x)$ up c units
$f(x) - c$	Shifts $f(x)$ down c units
$f(x + c)$	Shifts $f(x)$ left c units
$f(x - c)$	Shifts $f(x)$ right c units
$f(-x)$	Reflects $f(x)$ over the y-axis
$-f(x)$	Reflects $f(x)$ over the x-axis

To graph the equation of a function, first determine all the reflections, then determine the translations, and then graph.

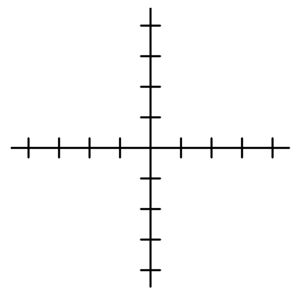
1. Graph the following functions. Then determine the domain and range for each function and provide them in interval form.

$f(x) = (x+1)^2 - 3$



Domain: Range:

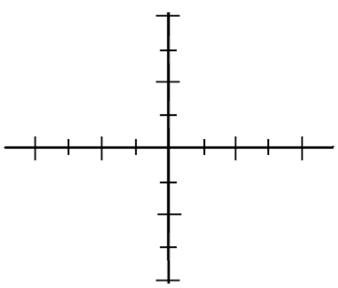
$f(x) = 2 - \sqrt{x-1}$



Domain: Range:

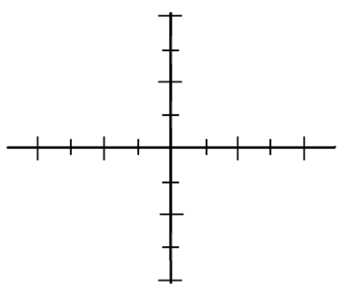
Stretches/Compressions

$f(x) = x^2$



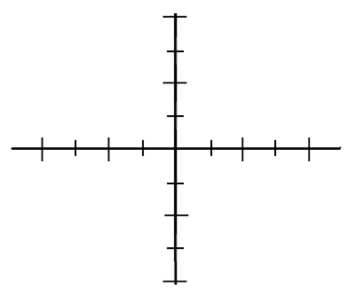
Vertical Stretch example:

$f(x) = 2x^2$



Vertical Compression example:

$f(x) = \frac{1}{2}x^2$



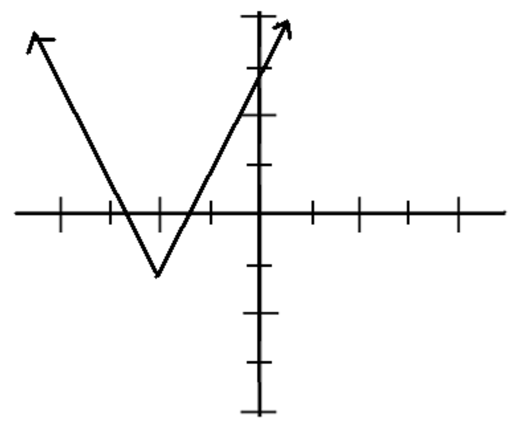
Given a function, $f(x)$, and a constant, c:

- If $cf(x)$, then $f(x)$ stretches vertically if $|c| > 1$ and compresses vertically if $|c| < 1$.
- If $f(cx)$, then $f(x)$ compresses horizontally if $|c| > 1$ and stretches horizontally if $|c| < 1$.

2. Provide a verbal description of the transformations applied to $f(x)$ to obtain $g(x)$.

$$f(x) = x^2$$
$$g(x) = 3 - 6(x-2)^2$$

3. Given the graph $f(x)$, answer the following questions:



i. What is the parent function?

ii. Is the graph shifted left or right? If so, by how many units?

iii. Is the graph shifted up or down? If so, by how many units?

iv. Is the graph reflected on the x-axis, y-axis, or not at all?

v. Is the graph stretched or compressed vertically or not at all?

vi. What is the equation for $f(x)$?

Even Functions

- Symmetric around the y-axis
- $f(x) = f(-x)$
- Replace x with $-x$ to get the same equation

Odd Functions

- Symmetric around the origin
- $f(x) = -f(-x)$
- Replace x with $-x$ *and* y with $-y$ to get the same equation

STUDY EDGE TIP

A chart to help remember symmetry:

x-axis y-axis origin

4. Given that $f(x)$ is an odd function and $f(5) = -3$, what is another point on $f(x)$?

5. Which of the following are odd functions?

   i. $f(x) = \frac{3x-4}{x}$
   ii. $y = x^3 + 4x$
   iii. $x^2 + y^2 = 25$

6. Which of the following functions are even?

i. $y = x^2 + 3x + 1$
ii. $y = \frac{1}{\sqrt[3]{x^2+4}}$
iii. $f(x) = |x + 3|$
iv. $f(x) = |x| + 3$

7. Given that $f(x)$ is even with $f(3) = 4$, $g(x)$ is one-to-one with $g(4) = -5$, and $h(x) = 1 - 3x$, find:

i. $(h \circ f)(-3)$

ii. $(g^{-1} \circ h)(2)$